

Parametrization of the generalized Pareto distribution

2013-05-05, Stig Rosenlund

For $x \geq 0$ we have

$$\textbf{Form 1: } F(x) = 1 - \left(1 - \gamma \frac{x}{\sigma}\right)^{\frac{1}{\gamma}}, \quad (\sigma > 0, \gamma \leq 0) \quad (1)$$

$$\textbf{Form 2: } F(x) = 1 - \left(\frac{\alpha}{\alpha + x}\right)^{\delta}, \quad (\alpha > 0, \delta > 0) \quad (2)$$

$$\textbf{Form 3: } F(x) = 1 - (1 + \beta x)^{-\frac{1}{\phi}}, \quad (\beta > 0, \phi > 0) \quad (3)$$

For $\gamma = 0$ we interpret F as the exponential distribution:
 $F(x) = 1 - e^{-x/\sigma}$.

Form 1 is according to **Nader Tajvidi** (1996). Form 2 is according to lectures notes at SU by **Björn Johansson**. Form 3 is according to **Stig Rosenlund** (2000).

For $\gamma > 0$ the following relations hold.

A. Parameters by Forms 2 and 3 in terms of those of Form 1

$$\alpha = -\frac{\sigma}{\gamma} \quad \beta = -\frac{\gamma}{\sigma}$$

$$\delta = -\frac{1}{\gamma} \quad \phi = -\gamma$$

B. Parameters by Forms 1 and 3 in terms of those of Form 2

$$\sigma = \frac{\alpha}{\delta} \quad \beta = \frac{1}{\alpha}$$

$$\gamma = -\frac{1}{\delta} \quad \phi = \frac{1}{\delta}$$

C. Parameters by Forms 1 and 2 in terms of those of Form 3

$$\begin{aligned}\sigma &= \frac{\phi}{\beta} & \alpha &= \frac{1}{\beta} \\ \gamma &= -\phi & \delta &= \frac{1}{\phi}\end{aligned}$$