

Bichsel-Straub's pseudo-estimator and credibility weighted mean in Rapp multiclass

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1. OPTIMALITY OF ESTIMATORS AND RAPP MULTICLASS

Section 4.2 in [Ohlsson and Johansson \(2010\)](#) deals with credibility estimators in multiplicative models. The variance component estimate $\hat{\sigma}^2$ of σ^2 for within-group variation is given in formula (4.26) and the estimate $\hat{\tau}^2$ of τ^2 for between-groups variation in formula (4.27). We have $E[\hat{\sigma}^2] = \sigma^2$ and $E[\hat{\tau}^2] = \tau^2 \geq 0$.

A base constant μ , initially considered known, appears in the model. In practice it is replaced with an estimate $\hat{\mu}$, just as the relativity products γ_i are replaced with estimates as stated at the end of these authors' section 4.2.1.

These estimates can be replaced by optimal estimators in theory. In practice $\hat{\sigma}^2$ cannot be improved in a distribution-free model. But $\hat{\mu}$, if given as a weighted average with exposure weights (like total claim frequency, mean claim or risk premium), and $\hat{\tau}^2$ can.

The multiclass analysis part of the programming language Rapp implements section 4.2 in [Ohlsson and Johansson \(2010\)](#) for claim frequency, mean claim and risk premium. For the latter the marginal-totals or the Tweedie equations can be used. For mean claim these authors' recommended method, namely the Γ -GLM application to claim severities, is implemented. The default exponent p is then 2.

Rapp has options to use either $\hat{\tau}^2$ or an improved estimate, and also to use either $\hat{\mu}$ = base factor or an improved estimate. We will give a background and describe the improved estimates.

In addition Rapp implements our own (quite different) multiclass method, which however will not be treated in the sequel.

2. OVERVIEW OF OPTIMAL ESTIMATORS

[De Vylder \(1996\)](#) III.Ch.3 gives a discussion for the ordinary Bühlmann-Straub model without a multiplicative model. The discussion is transferable

to the [Ohlsson and Johansson \(2010\)](#) application of credibility to multiplicative models. If the [Ohlsson and Johansson \(2010\)](#), section 4.2, model is specialized to one argument with one class, i.e. $R = (\# \text{ of } i) = 1$, the ordinary Bühlmann-Straub model is retrieved.

[De Vylder \(1996\)](#) gives a credibility-weighted statistic X_{zw} which in the Bühlmann-Straub model is a better estimate of μ than $\hat{\mu}$, unless they are equal as is the case for equal weights. A statistic A_{pseu} is the optimal estimate of τ^2 under the conditions of no contract excesses. See [De Vylder \(1996\)](#) III.Ch.3, section 3.4.7, where an unpublished work by Bichsel and Straub is cited and credited with A_{pseu} . Here excess = kurtosis = Fisher's coefficient γ_2 involving the 2:nd and 4:th central moments. Namely

$$e(X) = \frac{E[(X - \mu)^4]}{E[(X - \mu)^2]^2} - 3 \text{ for a random variable } X \text{ with } E[X] = \mu.$$

It is safe to say that A_{pseu} would almost always be better than $\hat{\tau}^2$.

3. DETAILS OF OPTIMAL ESTIMATORS

In Rapp Proc Taran multiclass the options Bsaps and Bsxxw are available. Here Bsaps implements A_{pseu} and Bsxxw implements X_{zw} . Before explaining how we will give a translation of the notation of [De Vylder \(1996\)](#) to [Ohlsson and Johansson \(2010\)](#) and express A_{pseu} and X_{zw} in the notation of the latter. An alternative notation for X_{zw} is given to clarify its meaning in the [Ohlsson and Johansson \(2010\)](#) context.

De Vylder	Ohlsson and Johansson
μ	μ
σ^2	σ^2
a	τ^2
X_{ww}	$\bar{\bar{Y}}_{...}$
X_{zw}	$\sum_{j=1}^J \frac{z_j}{z_1 + \dots + z_J} \bar{\bar{Y}}_{.j} = \bar{\bar{Y}}_{wzw}$
S^2	$\hat{\sigma}^2$
A	$\hat{\tau}^2$
A_{pseu}	$\frac{1}{J-1} \sum_{j=1}^J z_j \left(\bar{\bar{Y}}_{.j} - \bar{\bar{Y}}_{wzw} \right)^2$

Here A_{pseu} is a pseudo-estimator in the sense that the defining sum contains $a = \tau^2$, which A_{pseu} shall estimate. This means that A_{pseu} has to be

computed iteratively. Take a starting estimate for a , preferably the classical estimator $\hat{\tau}^2$. Compute A_{pseu} in the first iteration as the sum above with this estimate for a . Put this value as a in the sum and compute a new A_{pseu} in the second iteration. And so on. The algorithm converges fast.

Optimality of A_{pseu} assuming no contract excesses

Optimality means that A_{pseu} has the smallest variance of possible estimators. See [De Vylder \(1996\)](#) III.Ch.3, section 3.4.7, Theorem 18.

In sections 3.3.7 and 3.4.5 [De Vylder \(1996\)](#) also has clarifying discussions of the bias of the estimators A_{pseu} and $A = \hat{\tau}^2$ related to the non-negativity of $a = \tau^2$ and the fact that the estimators can furnish negative values.

Optimality of X_{zw}

See [De Vylder \(1996\)](#) III.Ch.3, section 3.4.4, Theorem 15.

4. RAPP IMPLEMENTATION

Option Bsaps for variance pseudo-estimator in Rapp

A_{pseu} is used in formula (4.24) of [Ohlsson and Johansson \(2010\)](#) for $\hat{U}_j$. For practical purposes this can be deemed to be an improvement for all cases.

Option Bsxzw for credibility weighted mean in Rapp

$\bar{Y}_{\text{wzw}}$ is used in formula (4.24) of [Ohlsson and Johansson \(2010\)](#) for $\hat{U}_j$. This is done by multiplying $\tilde{z}_j \frac{\bar{Y}_{\cdot j}}{\hat{\mu}}$ with $\bar{Y}_{\dots} / \bar{Y}_{\text{wzw}}$, where an estimate $\hat{\mu}$ of μ is used.

For one argument with one class, $R = (\# \text{ of } i) = 1$, it is immediate that this is an improvement. With $\gamma_1 = 1$ the estimate of μ used in (4.24) of [Ohlsson and Johansson \(2010\)](#) without option Bsxzw is $\hat{\mu} = \bar{Y}_{\dots}$, so that (4.24) is

$$\hat{U}_j = \tilde{z}_j \frac{\bar{Y}_{\cdot j}}{\bar{Y}_{\dots}} + (1 - \tilde{z}_j) \quad (1)$$

With option Bsxzw is used instead

$$\tilde{U}_j = \tilde{z}_j \frac{\bar{Y}_{\cdot j}}{\bar{Y}_{\text{wzw}}} + (1 - \tilde{z}_j) \quad (2)$$

and this is the optimal expression (though with z_j replaced by its optimal estimate) given in [De Vylder \(1996\)](#) III.Ch.3, section 3.4.4, Theorem 16.

In other cases there might be drawbacks due to the properties of the multiplicative solutions intertwined with credibility, e.g. a lack of robustness.

5. COMPARISON OF ESTIMATES

We use the bus case of section 4.5 in [Ohlsson and Johansson \(2010\)](#). Here GLM Poisson log link is combined with credibility. KUNDNR 145 was excluded. We include also the pseudo-estimator of τ^2 that was implemented in Rapp prior to April 2011. It was obtained from an iterative solution similar to the one for A_{pseu} , but with a direct term in $\hat{\sigma}^2$ whose coefficient was set to depend on the coefficients for $(\bar{Y}_{.j} - \bar{Y}_{\text{wzw}})^2$ in a certain way. This estimator has been removed from Rapp, since A_{pseu} is better. It is of some interest to see just how wrong it was. We denote it by $\tilde{\tau}^2$.

We compare the estimates for τ^2 . The estimates for σ^2 also differ when the backfitting algorithm of section 4.2.2 in [Ohlsson and Johansson \(2010\)](#) is used, with iterations between credibility and ordinary GLM. These estimates are not so interesting.

In Table 1 we state the results when the backfitting algorithm was not used, and in Table 2 when it was used with a maximum of five iterations. That was also the actual number of iterations.

TABLE 1. NO BACKFITTING

$\hat{\tau}/\hat{\mu}$	$\tilde{\tau}/\hat{\mu}$	$A_{\text{pseu}}^{\frac{1}{2}}/\hat{\mu}$
0.5407	0.5164	0.5172

TABLE 2. FIVE ITERATIONS

Bsxzw	$\hat{\tau}/\hat{\mu}$	$\tilde{\tau}/\hat{\mu}$	$A_{\text{pseu}}^{\frac{1}{2}}/\hat{\mu}$
Not used	0.5996	0.5363	0.5380
Used	0.7723	0.6253	0.6566

REFERENCES

- DE VYLDER, F. E. (1996) *Advanced Risk Theory*. Editions de l'Université de Bruxelles, Bruxelles.
- OHLSSON, E. and JOHANSSON, B. (2010) *Non-Life Insurance Pricing with Generalized Linear Models*. Springer, Berlin.

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